Building Portfolio Fragility Functions to Support Scalable Community Resilience Assessment

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Abstract: Community resilience planning, risk mitigation and recovery optimization must assume a system perspective at the level of the overall community built environment. While engineers can quantify the performance of individual buildings and facilities, such information must be aggregated to reflect the vulnerability of the building portfolio as a whole to support resilience-based decisions at the community level. Furthermore, such aggregated measurements are more effective in risk communication among social scientists, economists and other community decision makers than individual building measurements. This study presents a methodology for building portfolio analysis that relates the performance of individual buildings exposed to natural hazards to the overall performance of a building portfolio. We introduce the concept of building portfolio fragility function (BPFF), defined as the probability that a building portfolio, as an aggregated system, fails to achieve prescribed performance objectives (expressed in terms of social-economic metrics) conditioned on scenario hazards, to characterize the vulnerability of a building portfolio and to directly inform resilience-driven decisions at the community level. The paper concludes with an illustration of the development of BPFFs to the Centerville community.

Key words: community resilience; portfolio fragility function; risk; spatial correlation; system performance objectives.

1. Introduction

Communities require integrated community-level planning to achieve disaster preparedness and to mitigate risk from natural and man-made hazards. Codes and standards are necessary but not sufficient for community hazard protection and alleviation. In recent disaster events, such as Hurricane Katrina in 2005, Superstorm Sandy in 2012, and the Moore, OK tornado in 2013, communities suffered significant damages, economic losses and social disruptions despite the fact that buildings and infrastructure systems had been designed and constructed according to building regulations and accepted construction practices. To a considerable degree, this can be attributed to the fact that current building design codes and construction practices focus on safeguarding public life safety and robustness of individual buildings, and seldom consider the collective social-economic impact of severely damaged or functionally impaired building inventories on a community

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scale. To mitigate the impact of future natural hazards on communities, resilience planning must take on a system perspective at the level of the overall community built environment, thus comprehensively addressing the threat from disasters and also providing guidance and strategies for hazard mitigation and recovery optimization (Poland, 2013; OSSPAC, 2013).

The resilience of a community or a system, as illustrated conceptually in Figure 1, is often expressed in terms of its functionality, $Z$, as a function of time, and is associated with four attributes (Bruneau et al., 2003): robustness - the ability to withstand an extreme event and deliver a certain level of service even after the occurrence of that event; rapidity - to recover the desired functionality as fast as possible; redundancy - the extent to which elements and components of a system can be substituted for one another; and resourcefulness - the capacity to identify problems, establish priorities, and mobilize personnel and financial resources after an extreme event.

The community functionality, $Z(t)$, defined as the capability of a community to maintain its intended purposes and provide for the well-being of its citizenry, can be categorized as: technical—the ability of physical components and systems (e.g. buildings, transportation and utility networks) to withstand hazards; organizational—the capacity of various organizations that manage essential facilities (e.g., government and emergency response agencies) to plan, make decisions and take actions prior to, during and following the occurrence of a hazard event; social—the ability of people to design measures to lessen the negative social consequences; and economic—the capacity of community to reduce both direct and indirect economic losses from hazard consequences (Bruneau et al., 2003). These four interdependent dimensions of community functionality can be quantified by multi-dimensional metrics to collectively reflect the community’s resilience or lack of it. As a starting point, advanced systematic physical damage assessment for building portfolios and civil infrastructure networks is urgently required to evaluate technical functionality loss of a community; such models serve as the basis for evaluating the potential impact of hazard events on the organizational, social and economic functionalities and structures within a community.

Figure 1. Illustration of Resilience Concept
Significant uncertainties, both aleatory and epistemic, are associated with community functionality evaluation and have considerable impact on the community resilience assessment, as shown in Figure 1. Damage evaluation of individual buildings or other engineered facilities conventionally involves uncertainty modeling in hazard demand quantification, structural response assessment, damage prediction and loss estimation. Analysis of building portfolios is more complex due to spatial correlations between the random variables modeling the hazard demand at different building sites (i.e. site-to-site correlation) caused by the common hazard with a large footprint as well as spatial correlations between capacities of different buildings (i.e. structure-to-structure correlation) introduced by common structural design and construction practices. These correlations tend to be positive in nature, and failure to consider them is likely to result in unconservative errors in community functionality estimates and resilience assessments. Unfortunately, many current loss estimation tools, such as HAZUS-MH (FEMA/NIBS, 2003) and MAEViz (Steelman et al., 2007) do not consider these spatial correlations in risk assessments of spatially distributed building portfolios and infrastructure systems. Moreover, the computational effort involved in considering such correlations can be onerous, especially for large building inventories and utility networks. Many studies (e.g. Lee and Kiremidjian, 2007; Miller, 2011; Bonstrom and Corotis, 2015) have attempted to reduce this computational effort by assuming that the relevant random parameters are normally distributed and estimating only the first two orders of statistics of the interested portfolio performance metrics.

This study will focus on probabilistic estimation of functionality (or performance) metrics of spatially distributed community building portfolios for prescribed seismic scenario events, in which the building portfolio can represent a building block, a neighborhood, a group of essential facilities (e.g. hospitals and schools), a zone of commercial/retail facilities, or an entire community building inventory at various scales and resolutions, depending on the decisions to be made from the resilience assessment. First, a probabilistic estimation model for functionality of spatially distributed buildings, which incorporates uncertainties and correlations in both hazard demands and structure capacities, is outlined in detail. Second, to facilitate community-level resilience-based decisions and to establish the link between the technical and social-economic community functionality metrics, we propose a novel concept of building portfolio fragility functions (BPFF), defined as the probabilities that a building portfolio fails to achieve a threshold level of functionality (or portfolio performance objective) for a spectrum of considered scenario hazards. The BPFF aggregates performances (or damages) of individual buildings to obtain an estimate of the functionality of a building portfolio as a whole; it is based on engineering principles but is expressed in terms of portfolio functionality metrics that can directly inform resilience-driven decisions at the community level. Third, we develop a random sampling

1 The appropriate scenario event for resilience assessment of a real community should be identified based on the hazard frequency analysis and the risk tolerance of the specific community under investigation. For example, ASCE 7 (ASCE, 2010) is based on an earthquake (MCE) with a return period of 2475 years (2%/50 years). For Memphis, TN, for example, the dominant contributors to this return period hazard come from two faults that are 35 and 65 km from Memphis (Reelfoot Fault System), either of which is capable of generating scenario earthquakes of this magnitude.

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model to ensure computational efficiency in uncertainty propagation and analysis when dealing with building portfolios of different sizes and scales. The tradeoff between the resolution of sampling and the accuracy of the probabilistic portfolio analysis is also investigated. Finally, the paper concludes with an illustration of the building portfolio analysis framework developed herein using the Centerville community as a testbed.

2. Probabilistic Framework for Building Portfolio Analysis

2.2.1 Overall concept

Conceptually, the building portfolio risk assessment framework can be applied to different natural hazards. Using a scenario earthquake hazard for illustration, the functionality loss of a building portfolio due to its physical damage can be estimated probabilistically by quantifying the seismic hazard intensity at each building locations within the community, evaluating structural response and determining physical damage to each of the buildings (including structural and nonstructural components), and finally estimating portfolio functionality losses by establishing the functional relation between the collective performance of individual buildings and overall functionality of the portfolio as a whole. Each step of this process is conditioned on the previous step and can be carried out at different levels of detail, which may result in different sources of uncertainties being propagated through the risk assessment (Cornell and Krawinkler, 2000).

The distribution of the building portfolio functionality metric of interest, \( Z \), conditioned on an predefined earthquake scenario, \( S_{EQ} \), characterized by a specific magnitude \( M_w \) and epicenter distance \( D \), can be formulated as:

\[
P(Z \leq z | S_{EQ}) = \int_u \int_v F_{Z|DV}(z|u) f_{DV|DS}(u|v) f_{DS|IM}(v|y) f_{IM|S_{EQ}}(y|S_{EQ}) du dv dy \quad (1)
\]

where, reading from right to the left, \( f_{IM|S_{EQ}}(y|S_{EQ}) \) is the probability density function (PDF) of ground motion intensity \( IM \) conditioned on the scenario event \( S_{EQ} \); \( f_{DS|IM}(v|y) \) is the PDF of damage state \( DS \) conditional on \( IM \), which is often given by a fragility function, defined as the probability that the response of a building equals or exceeds a stipulated damage state as a function of hazard intensity; \( f_{DV|DS}(u|v) \) is the PDF of damage value \( DV \) conditioned on \( DS \); and \( F_{Z|DV}(z|u) \) is the cumulative distribution

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2 Ground motion intensity is the ground motion characteristic that can be related to the response of structural systems, nonstructural components, and building contents through engineering analysis, such as peak ground acceleration, peak ground velocity, peak ground displacement, or a spectral response quantity such as spectral displacement, velocity or acceleration.

3 The damage state categorizes the extent of damage to structural and nonstructural components by different damage levels (often related to the structural system deformation or acceleration). In HAZUS-MH (FEMA/NIBS, 2003), four damage states (i.e., slight, moderate, extensive, and complete) to structural and nonstructural components of a building and their relationship with building response threshold are identified.

4 The damage value is defined as the functionality loss to individual buildings with respect to the portfolio functionality of interest as a result of its physical damage. The damage values often are categorized as direct dollar losses, downtime (or restoration time), and deaths (causalities) (FEMA, 2012).
function (CDF) of portfolio functionality $Z$ conditional on $DV$. $IM$, $DS$ and $DV$ are all vectors and the dimension of these vectors is consistent with the number of buildings in the considered portfolio. Finally, $P(Z \leq z|S_{EQ})$, the CDF of $Z|S_{EQ}$, is given by the convolution of all the conditional probability distributions associated with these intermediate variables (i.e. $IM|S_{EQ}$, $DS|IM$, and $DV|DS$), which intermediately relate $Z$ to $S_{EQ}$.

At the individual building level, Eq. (1) reflects three types of uncertainties: 1) the uncertainty in ground motion intensity at the site of a building for a given earthquake scenario ($IM|S_{EQ}$), which mainly depends on the wave propagation path, local soil type and the selected attenuation model; 2) the uncertainty in structural or nonstructural damage of a building given the ground motion intensity ($DS|IM$), characterized by the discrete damage state probabilities which stem from the uncertainties in building capacity as well as in damage state definition; 3) the uncertainty in damage values conditioned on damage state ($DV|DS$), the conditional mean of which is often based on engineering judgement or collected empirical data. Note that Eq. (1) defines the probability of $Z$ for a scenario event; the uncertainties in the earthquake occurrence and fault location are not considered.

At the portfolio level, the aforementioned two types of correlations, i.e., the site-to-site correlation in hazard demand (captured in the vector $IM$) and the structure-to-structure correlation in building capacities (captured in the vector $DS$), are embedded in Eq. (1), reflecting the intrinsic characteristics of spatially distributed building portfolios. When these correlations between any building pairs are considered simultaneously for a building portfolio which may consist of thousands or even millions of buildings, Eq. (1) cannot be evaluated in closed-form. Moreover, a numerical solution using only Monte Carlo simulation (MCS) for the convolution is both challenging and costly due to the simulation of the three layers of conditional random fields and the high dimensions of correlated random variables in each layer. To address these problems, we take the following steps to evaluate Eq. (1): i) Generate $l$ sample vectors of $IM$ using MCS from the distributions of intensity measures at all building sites and the correlation structure among them for earthquake scenario $S_{EQ}$. Since earthquake demand is one of the most important sources of uncertainty in seismic risk assessment, MCS is utilized at this step to maintain the statistical characteristics of $IM$. ii) Approximate the mean and variance of metric $Z$ for a given sample vector $IM$, i.e., $Z|IM$. In this step, $Z|IM$, representing an aggregation (or summation) of the performances of individual buildings for a sample vector of $IM$, is assumed to be normally distributed based on the central limit theorem. Since the full probability distributions of the intermediate variables in Eq. (1) seldom are known, this assumption is acceptable (Baker and Cornell, 2008). This process is repeated $l$ times for each sample vector $IM$ generated in step i); iii) Finally estimate the distribution of $Z$ for the scenario event, i.e., $Z|S_{EQ}$, by using MCS again to generate $k$ samples from each of the $l$ normal distributions of $Z|IM$ obtained in step ii). Note that no assumption is made in this
step as to the distribution type of $Z | S_{EQ}$. The entire procedure is detailed next, starting with the modeling of correlations in Section 2.2.2.

2.2.2 Modeling of spatial correlations

Research studies in the past on damage and loss estimation for building portfolios (FEMA/NIBS, 2003) have treated individual building damages and losses as if they were statistically independent. Most previous researches attempting to model spatial correlations in seismic risk assessment have focused only on correlation in ground motion intensity, more specifically, on developing site-to-site correlation models for both intra- and inter-event, mainly based on the covariance analysis of typical ground motions at different sites (Adachi and Ellingwood, 2008; Goda and Hong, 2008; Jayaram and Baker, 2009; Miller, 2011; Bonstrom and Corotis, 2015). Only very few have investigated both site-to-site and structure-to-structure correlations, the latter originated from the similar design and construction practice and code enforcement (Lee and Kiremidjian, 2007; Vitoontus and Ellingwood, 2013). Such correlations depend on the demand from hazardous events over the affected area, the number of structures and their locations, and their susceptibility to damage if the hazardous event occurs. Since these correlations generally are positive, their neglect will invariably cause the second-order statistics (e.g. variance, confidence interval, upper fractiles, etc.) associated with the portfolio functionality metrics (e.g. direct losses, immediate occupancy, etc.) to be underestimated (see, e.g., Vitoontus and Ellingwood (2013) for a discussion of its impact on the determination of probable maximum losses due to earthquake).

The uncertainty of ground motion intensity at a site ($IM$) can be modeled with an intra-event error term $\xi$ and an inter-event error term $\eta$ (Jayaram and Baker, 2009); the latter is not needed for the scenario analysis herein. Accordingly, the ground motion intensity at building site $i$ can be written as:

$$\ln(IM_i) = \ln(IM_{\bar{i}}) + \tau \cdot \xi_i$$

where $IM_{\bar{i}}$ is the expected value of ground motion intensity at building site $i$ computed from a selected ground motion attenuation model; $\xi_i$ is often described by a standard normal distribution, and $\tau$ represents the standard deviation of $\ln(IM_i)$. The joint probability of ground motion intensity at all building sites is a multivariate lognormal distribution.

The site-to-site correlation in $\xi$ between two building sites $i$ and $j$, $\rho_{ij}^{IM}$, is often defined as an exponential function with respect to the separation distance between the two sites. For example, Goda and Hong (2008) proposed exponential correlations as functions of both separation distance between two buildings and their natural vibration periods. Jayaram and Baker (2009) concluded that the rate of decay in correlation typically decreases with increasing spectral acceleration period. For the study discussed herein, the correlation function determined by Wang and Takada (2005) is utilized:
where $r_{ij}$ is the separation distance between building sites $i$ and $j$; and $R$ is a parameter denoting the correlation distance, which is related to the characteristics of the earthquake and local site conditions.

The structure-to-structure correlation due to common construction material, regulatory practice and code enforcement has been discussed in only a few studies due to its complexity and the lack of available data specific to the system of interest. Vitoontus and Ellingwood (2013) modeled the damage state of building $i$, $DS_i$, as the sum of the effects of its construction material $M_i$, structural type $T_i$, and building code $C_i$, and a noise term $\varepsilon_i$:

$$DS_i = M_i + T_i + C_i + \varepsilon_i$$  \hspace{1cm} (4)

If

$$Y_i = M_i + T_i + C_i$$  \hspace{1cm} (5)

and $Y_i$ is statistically independent of $\varepsilon_i$, then the correlation between $Y_i$ and $Y_j$ and the correlation between $\varepsilon_i$ and $\varepsilon_j$ can be written as:

$$\rho_{Y_i,Y_j} = \rho_{MT_i,MT_j} \cdot \rho_{C_i,C_j}$$  \hspace{1cm} (6)

$$\rho_{\varepsilon_i,\varepsilon_j} = \exp \left( -\frac{r_{ij}}{\beta_\varepsilon} \right)$$  \hspace{1cm} (7)

where $\rho_{MT_i,MT_j}$ represents the correlation in responses of building $i$ and $j$ introduced by similar construction type and material; $\rho_{C_i,C_j}$, represents the same (or similar) building code; and $\beta_\varepsilon$ denotes the scale of the correlation due to building separation (buildings in proximity to one another are more likely to be highly correlated because of community development patterns). Since the damage value is functionally related to the damage state, the correlation between damage values of two buildings is assumed the same as that between damage states. Accordingly, the structure-to-structure correlation between building $i$ and building $j$ becomes:

$$\rho_{i,j}^{DS} = \rho_{i,j}^{DV} = \frac{\rho_{Y_i,Y_j} \cdot \sigma_{Y_i} \cdot \sigma_{Y_j} + \exp(-r_{ij}/\beta_\varepsilon) \cdot \sigma_\varepsilon^2}{\sqrt{\sigma_{Y_i}^2 + \sigma_\varepsilon^2} \cdot \sqrt{\sigma_{Y_j}^2 + \sigma_\varepsilon^2}}$$  \hspace{1cm} (8)

where $\sigma_{Y_i}$, $\sigma_{Y_j}$ and $\sigma_\varepsilon$ denote the standard deviation of $Y_i$, $Y_j$ and $\varepsilon$, respectively. $\sigma_{Y_i}$ and $\sigma_{Y_j}$ are computed as the root mean square of the logarithmic standard deviations of all damage values while the $\sigma_\varepsilon$ is often assumed to be a certain percentage of $\sigma_{Y_i} \cdot \sigma_{Y_j}$ in the literature (Vitoontus and Ellingwood, 2013).
2.2.3 Damage estimation of individual buildings conditioned on IM

The probabilistic estimate of damage to each building is the basis for calculating post-event portfolio functionality losses based on the mechanism by which individual buildings collectively support the portfolio functionalities of interest.

The $DV_i$ of building $i$ conditional on the $IM_i$ is:

$$f_{DV_i|IM_i} = \int_v f_{DV_i|DS_i}(u|v)f_{DS_i|IM_i}(v|y)dv$$

(cf Eq. (1)), and the subscript $i$ indicates that all terms in Eq. (9) pertain to building $i$. Since $DS_i$ represents discrete damage states, the mean and the standard deviation of $DV_i|IM_i$ are:

$$\mu_{DV_i|IM_i} = \sum_m \mu_{DV_i|DS_i} \cdot P(DS_i = ds_m|IM_i)$$

$$\sigma^2_{DV_i|IM_i} = \sum_m (\sigma^2_{DV_i|DS_i} + \mu^2_{DV_i|DS_i}) \cdot P(DS_i = ds_m|IM_i) - \mu^2_{DV_i|IM_i}$$

where $P(DS_i = ds_m|IM_i)$ is the probability of a particular damage state, $ds_m$, conditioned on the ground motion intensity at the site, $IM_i$, obtained from building fragility functions such as those illustrated in Figure 2(a). The random variable $DV_i|DS_i$, as shown in Figure 2b, can be described by different types of distributions depending the portfolio functionality of interest; $\mu_{DV_i|DS_i}$ and $\sigma^2_{DV_i|DS_i}$ are its mean and variance, respectively (Steelman et al., 2007; Baker and Cornell, 2008).

![Figure 2: Illustration of (a) CDF of $DS_i|IM_i$ and (b) PDF of $DV_i|DS_i$](http://www.tandfonline.com/doi/abs/10.1080/23789689.2016.1254997)

2.2.4 Aggregation of individual building damage to obtain probabilistic estimation of portfolio functionality metrics

The portfolio functionality metric $Z$ of a building portfolio is a function of the performance of individual buildings within the portfolio:
where $N$ is the number of buildings in the portfolio. As will be seen in Section 3, commonly recognized portfolio functionality metrics (e.g. portfolio direct loss and immediate occupancy ratio) are often linear functions of $DV$, as they represent a linear aggregation of $DV_i$ of individual buildings, i.e.

$$Z = \sum_{i=1}^{N} a_i \cdot DV_i + b$$

(13)

where $a_i$ and $b$ are constants. The mean and variance of $Z$, conditioned on $IM$, are:

$$\mu_{Z|IM} = \sum_i a_i \cdot \mu_{DV_i|IM_i} + b$$

(14)

$$\sigma_{Z|IM}^2 = \sum_i a_i^2 \cdot \sigma_{DV_i|IM_i}^2 + \sum_j \sum_i a_i \cdot a_j \cdot \rho_{i,j}^{DV} \cdot \sigma_{DV_i|IM_i} \cdot \sigma_{DV_j|IM_i}$$

(15)

in which $\mu_{DV_i|IM_i}$ and $\sigma_{DV_i|IM_i}^2$ are obtained from Eq. (10) and Eq. (11), respectively, and $\rho_{i,j}^{DV}$ is structure-to-structure correlation defined in Eq. (8). However, in the event that $Z$ is not a linear function, we can expand the Eq. (12) in a Tylor series and approximate its mean and standard deviation from the first-order terms in the expansion.

Finally, Eq. (1) becomes:

$$P(Z \leq z|S_{EQ}) = \int_y F_{Z|IM}(z|y) f_{IM|S_{EQ}}(y|S_{EQ}) dy$$

(16)

where $F_{Z|IM}(z|y)$ is the CDF of $Z$ conditioned on $IM$, which is assumed to be normally distributed according to central limit theorem, as discussed previously, with mean and variance computed by Eq. (14) and Eq. (15) or through a Tylor expansion of Eq. (13). The overall integration in Eq. (16), is performed by generating sample vectors of $IM$ using MCS according to the correlation structures defined in Eq. (3), estimating $Z|IM$ using Eqs. (9)-(15), for each sampled $IM$, and deriving the empirical distribution of $Z|S_{EQ}$ by taking an equal number of Monte Carlo samples from each of the normal distributions of $Z|IM$ for all samples $IM$. In this process, the functional relation between $Z$ and $DV$ (i.e. Eq. (12)) must be known; this relation is naturally different for different portfolio metrics $Z$ of interest.

3. Building Portfolio Functionality Metrics ($Z$)

Effective community functionality metrics should be scalable and actionable indicators of the community’s capacity to respond to and recover from a specified hazard (NIST, 2015), and should cover a broad spectrum of categories, ranging from physical damages to infrastructure, direct economic losses, recovery time, and social well-being. For example, Bruneau et al. (2003) identified a set of different community functionality metrics for various critical facilities based on the technical, organization, social, and economic dimensions of community resilience. Cutter et al. (2008) enumerated a broad category of typical community resilience indicators, e.g., wealth generation for economic development, and pointed out that identification of a common set of metrics covering environmental,
social, economic, institutional, infrastructural and community-based dimensions for assessing disaster resilience remains a challenge. Most community functionality metrics summarized in the literature are illustrative and conceptual in nature; ongoing research efforts, including those sponsored by NIST (2015), are expected to identify and collect information in the coming years to facilitate the quantification of those metrics and to support actionable community resilience planning and decision-making.

Following an extreme hazard event, the social-economical functionalities of a community are very much determined by the level of sustained physical damages of the building portfolios and other infrastructure networks within the community. Ability to shelter in place, direct economic loss and population dislocation are among commonly used metrics in the literature and are directly affected by the performance of the building portfolios in a community. In this study, these three building portfolio functionality metrics i.e. immediate occupancy ratio (IOR), household dislocation ratio (HDR), and direct monetary loss ratio (DLR) are utilized.

3.1. Immediate occupancy ratio (IOR)

Immediate occupancy ratio, IOR, is defined as the percentage of a building portfolio that can provide safe occupancy immediately following a disaster (Council, B. S. S., 1997). Whether a building can provide immediate shelter following a hazard event depends on the level of damage of the building. For earthquake hazard, four damage states ($DS$) are often considered: slight, moderate, extensive and complete (e.g. as in HAZUS), as illustrated in Figure 2. We assume that buildings without damage ($d_0$) or in damage states slight ($d_1$) or moderate ($d_2$) are still safe to occupy immediately following the event. However, when buildings exhibit extensive ($d_3$) or complete ($d_4$) structural damage, they become unsafe to occupy until they are repaired or replaced. Let the binary variable $IO_i$ denotes the status of immediate occupancy of building, $i$, with 1 being safe to occupy and 0 otherwise, that is:

$$ IO_i = \begin{cases} 
1, & \text{if } DS_i = ds_0, ds_1 \text{ or } ds_2 \ (\text{safe to occupy}) \\
0, & \text{if } DS_i = ds_3 \text{ or } ds_4 \ (\text{unsafe to occupy}) 
\end{cases} \quad (17) $$

Accordingly,

$$ IOR = \frac{\sum_{i=1}^{N} IO_i}{N} \quad (18) $$

The $DS_i$ of each building $i$ and associated probability as well as the correlation structure among $DS_i$ for all the buildings in the considered portfolio are the key parameters to determine the portfolio IOR. These parameters can be estimated using the methodology introduced in Section 2.

3.2. Household dislocation ratio (HDR)

Significant household dislocation is one of the most undesirable outcomes to a community following an extreme hazard event; it may be caused by a variety of reasons, with one of the major contributors being lack of habitable residential buildings. Household dislocation
ratio, HDR, is defined as the percentage of households in a community that are displaced due to loss of housing habitability and short-term shelter needs (FEMA/NIBS, 2003). The HDR is an important metric of community social vulnerability to natural hazard. In this study, we adopt the ordinary least squares (OLS) regression model (Peacock et al., 2016) to estimate the HDR:

$$\text{HDR} = \delta_{\text{vloss}} \times [b_1 + b_2 \cdot \delta_{\text{minorities}} + b_3 \cdot \delta_{\text{VAC}} + b_4 \cdot \delta_{\text{MHHIK}} + b_5 \cdot \delta_{\text{SFDET}}]$$  \hspace{1cm} (19)$$

where $\delta_{\text{minorities}}$ is the percentage of minorities in the community; $\delta_{\text{VAC}}$ is the percentage of vacant housing units; $\delta_{\text{MHHIK}}$ is the median housing income in the community (in $K$); $\delta_{\text{SFDET}}$ is the percentage of detached single family houses; and $b_i$ ($i = 1, 2, ..., 5$) are regression coefficients obtained using building portfolio data, population demographics, post-event damage and social survey data from past hazard events (Peacock et al., 2016; Lin, 2009; Girard and Peacock, 1997). $\delta_{\text{vloss}}$ is the fraction of direct losses (structural and non-structural component) of all buildings within a portfolio to the total portfolio replacement cost, which is a key parameter for determining the portfolio HDR and can be estimated from $DV_i$ presented in Section 2.

### 3.3. Direct loss ratio (DLR)

Monetary loss to a building inventory due to its physical damage, referred as the direct loss herein, is one of the most commonly studied building portfolio metrics. The direct loss ratio, DLR, defined as the ratio of total direct loss to total assessed value (including building contents) of a building portfolio, is often used for community-level policy making and insurance underwriting.

The direct loss of a portfolio can be computed as (Steelman et al., 2007):

$$\text{Loss} = \sum_{i=1}^{N} \text{Loss}_i = \sum_{i=1}^{N} M_i \cdot (a_i^{SD}DV_i^{SD} + a_i^{ND}DV_i^{ND} + a_i^{NA}DV_i^{NA} + a_i^{CL}DV_i^{CL})$$  \hspace{1cm} (20)$$

where $\text{Loss}_i$ is the direct loss of the building $i$; $M_i$ is the replacement cost of the building $i$; $a_i^{SD}$, $a_i^{ND}$, $a_i^{NA}$ are the fractions of the values of structural components, and non-structural acceleration-sensitive and drift-sensitive components$^5$, respectively; and $a_i^{CL}$ is the ratio of the contents value to the replacement cost. These values of $\alpha$ are usually determined from historical or empirical data collected by construction companies. In this study, these values are taken from the HAZUS-MH MR2 Technical Manual (FEMA/NIBS, 2003). $DV_i^{SD}, DV_i^{ND}, DV_i^{NA}$ and $DV_i^{CL}$ are the damage values in building $i$ of the above-mentioned components, which are random variables and can be assessed using the building portfolio analysis procedure outlined in Section 2. DLR can then be calculated as:

$$\text{DLR} = \frac{\sum \text{Loss}_i}{\sum_{i=1}^{N} M_i}$$  \hspace{1cm} (21)$$

$^5$ The structural components refer to the main load-resisting system; In HAZUS, the nonstructural components are grouped as either "drift-sensitive" or "acceleration-sensitive", in order to assess their damage due to an earthquake.
Table 1: Summary of building portfolio functionality metrics

<table>
<thead>
<tr>
<th>Portfolio Metrics</th>
<th>Abbrev.</th>
<th>Definition</th>
<th>Eqs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Immediate occupancy ratio</td>
<td>IOR</td>
<td>the percentage of a building portfolio that can provide safe occupancy immediately following a disaster</td>
<td>Eqs.(17);(18)</td>
</tr>
<tr>
<td>Household dislocation ratio</td>
<td>HDR</td>
<td>the percentage of displaced households in the community due to loss of housing habitability and short-term shelter needs</td>
<td>Eq. (19)</td>
</tr>
<tr>
<td>Direct loss ratio</td>
<td>DLR</td>
<td>the ratio of total direct loss to total assessed value of a building portfolio</td>
<td>Eqs.(20);(21)</td>
</tr>
</tbody>
</table>

The IOR, HDR and DLR, as summarized in Table 1, are all portfolio-level performance metrics aggregated from damage of individual buildings. These metrics are highly uncertain due to the epistemic and aleatory uncertainties involved in the intermediate engineering and social variables as well as the high-dimensional correlations between them, which can be captured through the building portfolio analysis procedure outlined in Section 2. In the next Section, we discuss how these metrics and associated uncertainties can best inform community decision makers and stakeholders.

4. Building Portfolio Fragility Function (BPFF)

To facilitate community-level resilience-based decisions we introduce the concept of building portfolio fragility functions, BPFF, defined as the probability that a building portfolio functionality metric \( Z \) (e.g. IOR, HDR or DLR) fails to achieve a prescribed portfolio performance objective \( L_{S_z} \), expressed in terms of a limiting value of metric \( Z \), conditioned on a spectrum of scenario earthquakes.

Figure 3 illustrates the procedure for developing the BPFF. Figure 3(a) plots the PDF of a portfolio functionality metric of interest, e.g. \( Z = HDR \), given the occurrence of, say, a \( M_w \) 8.0 scenario earthquake with an epicenter distance of 50km from the center of a community. This PDF of \( Z | S_{EQ} ; (M_w, 8.0, 50km) \) can be developed following the procedure in Sections 2 and 3. The shaded area is the probability that HDR exceeds a particular threshold, say 10%, i.e. \( L_{S_Z} = 0.1 \). Figure 3(b) shows a suite of three BPFFs associated with HDR exceeding a threshold value of 10%, 30% and 50% (i.e. \( L_{S_Z} = 0.1, 0.3 \) and 0.5), respectively. The circle on the BPFF with \( L_{S_Z} = 0.1 \) in Fig. 3(b) corresponds to the shaded area in Figure 3(a). The BPFF depicted in Figure 3(b) expresses the probabilities of exceeding three threshold values of a functionality metric as a function of earthquake magnitude, \( M_w \), for earthquakes emanating from the same seismic source and a constant epicentral distance of 50km. We find that such a BPFF can usually be approximated by a lognormal function, i.e.,

\[
P(Z > L_{S_Z} | S_{EQ}) = \Phi \left( \frac{\ln(M_w/\theta)}{\beta} \right)
\]  

(22)
in which the coefficients $\theta$ and $\beta$ can be estimated by curve-fitting through the exceeding probabilities with respect to a fixed threshold value of $LS_Z$ for a spectrum of interested scenario events.

Figure 3: Illustration of the development of BPFF: (a) PDF of portfolio functionality conditioned on a scenario earthquake and (b) BPFFs with respect to different portfolio functionality thresholds for a spectrum of scenario earthquakes

The portfolio functionality threshold, $LS_Z$, is considered as a limiting value for portfolio functionality, the occurrence of which would lead to unacceptable adverse impact on a community’s ability to function and to recovery within a target timeframe. Note that what is “unacceptable” is different from community to community; ultimately, it is up to a community to determine the goals that are most appropriate, and ideally, it should be developed by a diverse group of community stakeholders in a transparent, public process in order to properly address a potentially wide range of competing objectives and considerations (SPUR, 2009; Mieler et al., 2015). Specified building portfolio performance objectives would enable community decision makers to evaluate the gap between the target and the existing portfolio performances, and to develop actionable plans for hazard mitigation and infrastructure management in order to close this gap (Wang and Ellingwood, 2015; Lin et al., 2016).

The concept of the BPFF is similar to that of the traditional building fragility function in the sense that they both represent the conditional probability of exceeding a specified performance threshold for a given hazard. The traditional fragility function depicts the uncertainty in performance of a single building (e.g. with respect to inter-story drift), while the BPFF depicts the uncertainty in performance of a building portfolio with respect to technical, social or economic functionality limit states (e.g. with respect to IOR, HDR and DLR) established by the community for resilience planning and risk-based mitigation. The BPFF bridges the gap between the physical damage estimation of individual buildings and their collective impact on social-economic metrics that can directly inform community
resilience planning, and facilitate risk communication between engineers and other community-level decision makers, e.g., government officials, private stakeholders, etc.

5. Random Sampling Model of Community Building Portfolio

Despite the simplifications made to facilitate the evaluation of Eq. (1), the required computational effort to develop a BPFF, as summarized in Sections 2 through 4, is significant. When the number of buildings \( N \) is large (e.g. Shelby County, TN, has more than 300,000 buildings), the determination of \( \rho_{ij}^{IM} \) and \( \rho_{ij}^{DV} \), \( N \times N \) correlation matrices, is onerous. Moreover, the procedure for estimating portfolio functionality metric \( Z \) must be repeated for a sufficient number of \( IM \) with respect to a given scenario; and then the whole procedure must be repeated over for a spectrum of interested scenario events. To ensure the scalability of this BPFF framework to communities of different sizes, a random sampling technique is implemented, i.e. using a Poisson random field with \( n \) random samples of buildings to represent the entire building portfolio of \( N \) buildings, where \( n \ll N \). The choice of sampling size \( n \) depends on the size and topology of the building inventory, as well as the level of the site-to-site and structure-to-structure correlations (Vitoontus and Ellingwood, 2013).

To validate this sampling technique, we analyzed a 4km \( \times \) 1.5km non-homogenous residential zone in the U.S. of 4246 typical residential buildings of three types: 2196 non-seismically designed one-story wood frames developed mainly in the 1950s (denoted herein as “pre-code”, W1); 2000 seismically designed one-story wood single family dwellings developed during the 1970–1980s (“low-code”, W2), and 50 seismically designed wood residential buildings developed in 1990s (“moderate-code”, W3). Buildings of the same type developed in a similar period are located in a cluster, representing typical urban development patterns, and within each cluster, they are randomly scattered modeled by Poisson random fields, as shown in Figure 4(a); the random sampling applied to each cluster is illustrated in Figure 4(b). The relative proportion of the three types of buildings is maintained during the sampling process.

Figure 5 displays the empirical CDF of the IOR of this residential zone for a given scenario earthquake using different sampling sizes, \( n \), ranging from 20 to 2000. When \( n \) increases to 100, the estimated IOR gradually converges to the “exact” solution which involves all 4246 buildings. To measure the impact of correlation structures on the accuracy of the approximation using this sampling technique, Table 2 summarizes the relative error of the approximation associated with different correlation distances and sample sizes, with the relative error defined as:

\[
e = \frac{||\hat{z} - z||}{||z||}
\]

where \( \hat{z} \) is the approximation while \( z \) is the “exact”. In this particular case, the sample size \( n \) necessary to ensure accuracy in approximation is not much affected by the correlation distance when \( n \) is greater than 100. This sampling technique will be further implemented in a comprehensive case study in the following section.


http://www.tandfonline.com/doi/abs/10.1080/23789689.2016.1254997
Figure 4: Illustration of (a) a hypothetical residential zone and (b) random sampling of the houses in this zone.

Figure 5: Convergence of the random sample model using IOR as a portfolio functionality metric.
Table 2: Relative error associated with random sampling for different correlation distances and sample sizes

<table>
<thead>
<tr>
<th>Correlation distance</th>
<th>Sample size, n</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>20</td>
</tr>
<tr>
<td>R=2</td>
<td>0.0356</td>
</tr>
<tr>
<td>R=10</td>
<td>0.0281</td>
</tr>
<tr>
<td>R=20</td>
<td>0.0192</td>
</tr>
<tr>
<td>R=50</td>
<td>0.0377</td>
</tr>
<tr>
<td>R=100</td>
<td>0.0457</td>
</tr>
</tbody>
</table>

6. Centerville Building Portfolio Analysis

To demonstrate the implementation of the proposed building portfolio analysis procedure (in Section 2), the concept of the BPFF (in Section 4) and the random sampling technique (in Section 5), a case study is performed to the building portfolio in the Centerville Virtual Community. The IOR, HDR and DLR (introduced in Section 3) are adapted as building portfolio functionality metrics, for illustration.

The Centerville Virtual Community is described in detail elsewhere (Ellingwood et al., 2016). It is a typical mid-size community, with a population of approximately 50,000, situated in a Midwestern State in the US, and approximately 8km by 13km (5 miles by 8 miles) in size. The Centerville building portfolio contains approximately 30,000 buildings in 7 residential zones, 2 commercial zones, 2 industrial zones (1 light industry and 1 heavy industry), as shown in Figure 6(a). The 7 residential zones are categorized and distributed according to the income level of the residents as well as the building density. The 2 commercial zones are located along major roadways. The light industrial zone is located at the north of the community while the heavy industrial zone is located at the south east of the community, both along a railway for easy cargo transportation.

The Centerville building portfolio consists of 16 building types, which include residential, commercial, industrial occupancies, as well as critical facilities such as hospitals, fire stations, schools and government offices. The spatial distribution of all building types within Centerville is shown in Figure 6(b), while their detailed descriptions are presented in Table 3.
Figure 6: Centerville (a) zoning map, and (b) building portfolio
Table 3 Summary of Centerville building types

<table>
<thead>
<tr>
<th>BuildingTypeID</th>
<th>Construction</th>
<th>OccupancyClass</th>
<th>Story</th>
<th>YearBuilt</th>
<th>Area (ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>Wood</td>
<td>Residential_SF</td>
<td>1</td>
<td>1945-1970</td>
<td>1400</td>
</tr>
<tr>
<td>W2</td>
<td>Wood</td>
<td>Residential_SF</td>
<td>1</td>
<td>1985-2000</td>
<td>2400</td>
</tr>
<tr>
<td>W3</td>
<td>Wood</td>
<td>Residential_SF</td>
<td>2</td>
<td>1985-2000</td>
<td>3200</td>
</tr>
<tr>
<td>W4</td>
<td>Wood</td>
<td>Residential_SF</td>
<td>1</td>
<td>1970-1985</td>
<td>2400</td>
</tr>
<tr>
<td>W5</td>
<td>Wood</td>
<td>Residential_MF</td>
<td>3</td>
<td>1985</td>
<td>36000</td>
</tr>
<tr>
<td>W6</td>
<td>Wood</td>
<td>Mobile_Home</td>
<td>NA</td>
<td>NA</td>
<td>NA</td>
</tr>
<tr>
<td>S1</td>
<td>Steel braced frame</td>
<td>Commercial</td>
<td>1</td>
<td>1980</td>
<td>50000</td>
</tr>
<tr>
<td>RC1</td>
<td>RC frame</td>
<td>Commercial</td>
<td>2</td>
<td>1980</td>
<td>50000</td>
</tr>
<tr>
<td>RM1</td>
<td>Reinforced masonry</td>
<td>Commercial</td>
<td>2</td>
<td>1960</td>
<td>25000</td>
</tr>
<tr>
<td>S2</td>
<td>Mix of steel and OWSJ roof</td>
<td>Commercial</td>
<td>1</td>
<td>NA</td>
<td>125000</td>
</tr>
<tr>
<td>S3</td>
<td>Steel braced frame</td>
<td>Industrial</td>
<td>2</td>
<td>1975</td>
<td>100000</td>
</tr>
<tr>
<td>S4</td>
<td>Steel braced frame</td>
<td>Industrial</td>
<td>1</td>
<td>1995</td>
<td>500000</td>
</tr>
<tr>
<td>RC2</td>
<td>RC frame</td>
<td>Hospital</td>
<td>4</td>
<td>1980</td>
<td>120000</td>
</tr>
<tr>
<td>RM2</td>
<td>Reinforced masonry</td>
<td>Fire Station</td>
<td>2</td>
<td>1985</td>
<td>10000</td>
</tr>
<tr>
<td>RC3</td>
<td>RC frame</td>
<td>School</td>
<td>3</td>
<td>1990</td>
<td>100000</td>
</tr>
<tr>
<td>RM3</td>
<td>Light reinforced masonry</td>
<td>School</td>
<td>1</td>
<td>NA</td>
<td>100000</td>
</tr>
</tbody>
</table>

A hypothetical scenario earthquake with $M_w$ 7.8 and an epicenter located approximately 40 km southwest of Centerville is considered for illustration. The ground motion attenuation model by Campbell (2003) and the capacity spectral method are used to determine the IM at each building site, i.e., the spectral displacement $S_d$ for determining damage of structural and drift-sensitive nonstructural components, and spectral acceleration $S_a$ for acceleration-sensitive nonstructural components and building contents. Centerville is assumed, for simplicity, to be situated on Site Class B soils (ASCE 7-10). Seismic fragility functions for these above-mentioned components are mapped from the HAZUS-MH database (FEMA/NIBS, 2003), based on building characteristics such as occupancy, structural type, construction material, number of stories, square footage area and year built, to support the analysis herein. For assessing total direct loss ratio, DLR, empirical data from MAEViz database (Steelman et al., 2007) are adopted.

Both site-to-site and structure-to-structure spatial correlations are modeled in the probabilistic building portfolio analysis. The spatial correlation in seismic intensity between building sites is estimated by Eq. (3) assuming the correlation distance $R = 10$ km. This correlation distance value has been assumed to be in the range of 20-40 km (Wang and Takada, 2005), and it is scaled down herein since Centerville is a relatively small community. The standard deviation of intra-event error term $\tau$ in Eq. (2) is computed based on Campbell’s ground motion attenuation model (Campbell, 2003). The spatial correlations between $DVs$ of any two buildings are determined from Eq. (8), assuming that $\rho_{Y_i,Y_j} = 0.35$ when buildings $i$ and $j$ are of different building types, and $\rho_{Y_i,Y_j} = 0.9$ if otherwise. Ideally, for a “real” community, these correlation coefficients should be determined based on collected building inventory data coupled with professional judgement. The noise term $\sigma_\epsilon$ in Eq. (4) is neglected due to a lack of empirical data. The

three functionality metrics introduced in Section 3 are investigated for Centerville. The coefficients required for estimating these metrics are obtained from the original references of their formulation (such as HAZUS, MAEViz), which were mostly based on readily available data, either from collected databases or expert opinions.

The mean loss distribution within the Centerville building portfolio is illustrated in Figure 7. The heaviest losses, in terms of dollar losses, tend to occur in commercial/retail and industrial areas as shown in Figure 7(a), while in terms of DLR, they tend to occur in multifamily and high income residential zones. Aggregating these distributed losses within each zone, Figure 8 compares the mean direct dollar losses and the loss ratios between building zones. While Zone 11 (heavy industrial) represents the highest dollar losses, Zone 9 (retail/commercial) represents the highest economic impact in terms of DLR. In particular that Zone 7 (mobile home park) indicates a relatively low dollar value, but is among the highest in terms of DLR. In general, for all buildings, dollar losses due to the damage to structural components are much less than the losses due to non-structural components and structural contents.

Figure 9(a), (b) and (c) plot the complementary cumulative distribution functions of IOR, HDR and DLR, respectively, for Zone 1 (Z1), Zone 3 (Z3) and Zone 7 (Z7), representing high-, medium-, and low-income household zones, respectively, for the considered scenario event. The performance of Z7 with respect to IOR is much less favorable than that of Z1 and Z3, as shown in Figure 8(a). This is easily explained by the fact that mobile homes in Z7 generally experience more severe damage than the typical wood residential buildings in the other two zones; moreover, buildings in Z1, which are occupied by high-income households, are usually better constructed and likely to have better structural performance. For example, this figure indicates that the probability that more than 80% of buildings in Z1, Z3, and Z7 can provide immediate occupancy following the scenario earthquake is 55%, 38% and 5%, respectively. This information can inform community-level resilience-based risk mitigation strategies, e.g. selection of sites for temporary shelter; pre-event building portfolio retrofit decisions regarding spatial distribution of retrofit resources and choice of retrofit schemes, etc. (Lin et al., 2016).

Even with higher IOR, Figure 9(b) indicates, somewhat counter-intuitively, that the high-income Z1 also exhibits a higher HDR, reflecting the fact that wealthy households are more apt to dislocate because they are less tolerant to building damages and are more likely to have the necessary resources for relocation (Peacock et al., 2016). The complementary cumulative distributions of DLR, shown in Figure 9(c), indicate that the median loss for Z1, Z3 and Z7 is 8.4%, 6.9%, 10%, respectively. Such information is most informative for insurance underwriting and government subsidies policy-making.

The BPFFs with respect to different portfolio metrics as well as different threshold levels of these metrics are then developed for a spectrum of scenario hazards. To achieve computational efficiency, the random sampling model discussed in Section 5 is implemented with sample size \( n = 200 \). Figure 10(a) illustrates BPFFs for Z1 with respect to four levels of threshold values of IOR. The figure reads, for example, the probability that Z1 has less than 80% inhabitable houses following a \( M_w \)6.0, \( M_w \)7.0 or \( M_w \)8.0 earthquake (assume the same epicenter distance) is 1%, 19.5% and 65.7%, respectively.
Figure 10(b) compares BPFFs of Z1, Z3 and Z7 for a common threshold value of IOR = 80%, showing that Z7 is more vulnerable than Z1 and Z3 in terms of capacity to provide immediate occupancy. More importantly, the disparity in vulnerability in these zones increases as the seismic hazard intensity increases.

While the information presented above can be very useful for community resilience planning, the illustration required numerous assumptions in modeling correlations and commuting functionality metrics. Sensitivity studies should be performed to identify and guide the acquisition of the most important data for risk-informed decisions and to support the implementation of the proposed framework.

7. Summary and Conclusions

Community resilience planning, risk mitigation and recovery optimization must assume a system perspective at the overall community built environment. In this paper, a framework for community-level building portfolio functionality (or performance) analysis has been illustrated, in which the performance of individual buildings is aggregated to achieve community-level measurements of the performance of building portfolios. The concept of a BPFF is proposed which characterizes the vulnerability of a building portfolio as an integrated system. The BPFF is based on engineering principles but is expressed in terms of system-level technical, social and economic metrics, and therefore facilitates risk communication between engineers and other community-level decision makers (e.g., government officials, private stakeholders, etc.). The building portfolio analysis framework outlined herein when coupled with the random sampling algorithm is proved to be scalable and computationally efficient to be implemented to communities with different sizes and can effectively support community modeling and resilience-based decisions at different resolutions.

8. Acknowledgement

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Figure 7: (a) Expected direct loss and (b) expected direct loss ratio for each building in Centerville
Figure 8: (a) Expected direct loss and (b) expected loss ratio for each building zone and critical facility in Centerville.
(a) Immediate occupancy ratio (IOR)

(b) Household dislocation ratio (HDR)

Figure 9: Probability of exceeding (a) immediate occupancy ratio (IOR), (b) household dislocation ratio (HDR), and (c) direct loss ratio (DLR), for Centerville Zones 1, 3 and 7 for the Mw7.8 earthquake
(a) Z1 for four different threshold values

(b) Z1, Z3, and Z7 for a common threshold values of IOR=80%

Figure 10: BPFFs with respect to IOR for (a) building Zone 1 for four different threshold values, and (b) building Zone1, Zone3 and Zone 7 for a common threshold values of IOR=80%

References


